Mountain Waves and CAT Encountered by the XB-70 in the Stratosphere

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The data from 36 XB-70 flights conducted over the mountainous regions of the western United States together with rawinsonde data were used to investigate relationships between conditions favorable for mountain waves and clear air turbulence. Profiles for the Scorer parameter and the gradient Richardson number were evaluated from the rawinsonde data. The Scorer parameter and the gradient Richardson number profiles were computed on those days when the XB-70 flew, and these results compared to model profiles and related to the reported turbulence. Ascent rate profiles of rawinsonde balloons were analyzed from which the presence of mountain or lee waves was inferred. From the results of this investigation, objective methods were developed for forecasting days when turbulence in the stratosphere due to mountain waves should or should not occur, but not the specific location.

Introduction

CLEAR air turbulence (CAT) remains a major problem in aviation in spite of the number of studies made during the past two decades. A new dimension to the problem has been added by the supersonic transport which cruises between Mach 2 and 3 and at altitudes in excess of 70,000 ft (21 km). At these altitudes in the stratosphere, the problem is more difficult to attack than in the troposphere because of a reduction in both the quantity and quality of meteorological data.

There are a number of mechanisms which may lead to CAT. Some of these include dynamic instability[1] characterized by small Richardson numbers (Ri), breaking Kelvin-Helmholtz waves,[2] increased wind shear caused by vertical motion which leads to a lower Ri,[3] and lee or mountain waves.[4] Foltz[4] and others have shown that lee waves may lead to CAT, and that the severity of CAT is a function of the amplitude and wavelength of the lee waves. Moreover, it is known that lee waves may extend upward to heights exceeding 50 km.[5] The vertical propagation of wave energy has been considered by Hines and Reddy,[6] Palm and Foldvik,[7] Eliassen and Palm,[8] Booker and Bretherton,[9] and others. While it is beyond the scope of this paper to consider the mechanisms which lead to the vertical propagation of wave energy, these authors have established the fact that such does occur.

Data from XB-70 flights over the mountainous regions of the western United States together with rawinsonde data were used to investigate relationships between mountain-wave conditions and CAT. It is shown that conditions in the troposphere favorable for the development of lee waves are associated with CAT encountered by the XB-70 in the stratosphere. Also, a simple forecasting method for CAT in the stratosohere between 40 and 70K ft (12 and 21 km) is presented. The method is based on the formation of lee waves, and distinguishes between turbulent and nonturbulent conditions for about 80% of the cases considered, but without regard to specific location. The investigation is limited to the synoptic scale since rawinsonde was the only meteorological data available corresponding to the aircraft data.

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Mountain Waves and CAT

Scorer[10,11] showed that for dry, streamline flow which is inviscid and isentropic, lee waves should form when the parameter L^2 defined by

$$L^2 \equiv g\beta/U^2 - U''/U \tag{1}$$

decreased sufficiently with height. In this equation, L^2 is the Scorer parameter, β is a stability factor given by $(1/\bar{\theta})\partial\bar{\theta}/\partial z$ where $\bar{\theta}$ is average potential temperature, z is height, g is the acceleration of gravity, U is the horizontal wind speed, and U'' is the derivative with respect to height of the vertical wind shear (a second derivative).

The gradient Richardson number, Ri, defined by

$$Ri \equiv \frac{(g/\bar{\theta})\partial\bar{\theta}/\partial z}{(|\partial\bar{\mathbf{V}}/\partial z|)^2} \tag{2}$$

has been associated with CAT by many investigators, although the association is of dubious value in many cases. In this equation, $\bar{\mathbf{V}}$ is the average vector horizontal wind, and the other parameters have the same meaning as above. From theory, when Ri becomes less than some critical value, CAT should be generated.

Gazzola[12] summarizes atmospheric conditions favorable for the development of gravity waves resulting from orographic effects. Variables include terrain features, temperature stratification, and the distribution in the vertical of the horizontal wind. A temperature stratification favorable for the development of wave motions includes: 1) A layer of low stability near the ground, 2) a very stable layer above the surface layer, and 3) a low stability layer above the very stable layer. A vertical wind profile favorable for wave development contains: 1) A component of wind normal to the crest of the mountains with a minimum magnitude between 7 and 15 m sec^{-1}, 2) little variation of the wind direction with height, and 3) a gradual increase of wind speed with height into the upper troposphere. These conditions occur fequently when a properly oriented jet stream or zone of maximum wind moves across a mountain chain.

Wave motions may degenerate into turbulence through the effects of vertical motion (positive or negative) which

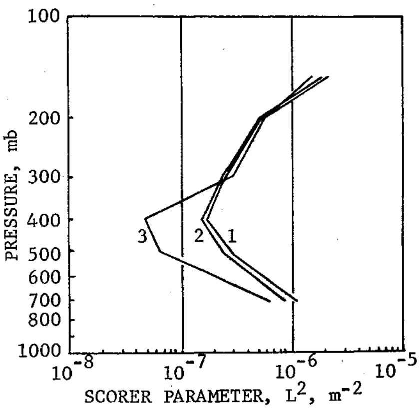


Fig. 1 The profiles of L^2 for the strong wind profile in combination with selected temperature profiles. (1-Strong wind and relatively stable temperature; 2-Strong wind and standard atmosphere; 3-Strong wind and relatively unstable temperature.)

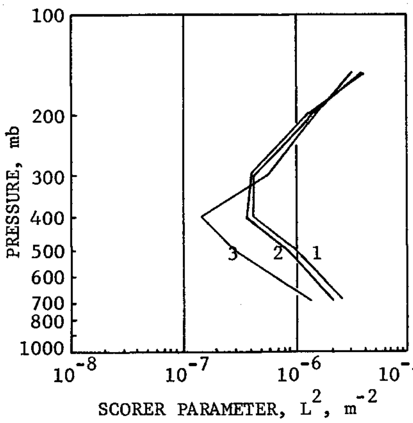


Fig. 2 The profiles of L^2 for the light wind profile in combination with selected temperature profiles. (1-Light wind and relatively stable temperature; 2-Light wind and standard atmosphere; 3-Light wind and relatively unstable temperature.)

may lead to a decrease in Ri[3,13,14] or by overturning of the wavecrest (Kelvin-Helmholtz instability) resulting in cold air sinking through a warm environment.[13] These processes lead to changes in wind shear and stability and, hence, in L^2 and Ri.

To illustrate the importance of wind shear and stability (lapse rate of temperature given by $\partial T/\partial Z$) on profiles of L^2 and Ri, several model profiles were constructed from which it is shown that lee waves and CAT should occur simultaneously, i.e., conditions that lead to a rapid decrease in L^2 with height also lead to a small Ri. Two model wind profiles were synthesized, the first having strong wind speeds and large vertical shears, while the second had light wind speeds and small vertical shears. In addition, three model temperature profiles were synthesized. The first was obtained from the 1962 U.S. Standard Atmosphere and used as a reference to develop two other temperature profiles. The second profile had a lapse rate of temperature between all levels that was less than (more stable) the corresponding lapse rate of temperature for the Standard Atmosphere, while the third profile had a lapse rate greater than that for the Standard Atmosphere (less stable). In effect, the relatively unstable temperature profile displays a lower tropopause than the other temperature profiles. For the corresponding pressure levels, identical wind directions were used for all wind profiles, and the heights selected for the pressure levels were those of the Standard Atmosphere. Therefore, variations among the values of L^2 and Ri for these wind and temperature combinations will be due solely to the effects of wind speed, wind shear, and stability. The complete data for the model profiles of wind and temperature are shown in Table 1. Using these data, Eqs. (1) and (2) were evaluated by use of centered finite differences. Computations were performed over two data intervals (Col. 1, Table 1), thus the derivatives are highly smoothed (averaged).

Each wind profile was used in combination with each of the three temperature profiles, and corresponding profiles of L^2 and Ri calculated. The profiles of L^2 are shown in Fig. 1 for the strong wind data in combination with each temperature profile, and in Fig. 2 for the light wind data in combination with each temperature profile. For each profile in Fig. 1, there is a decrease of about one order of magnitude or more in the value of L^2 from the 700-mb level to the minimum point of the profile at the 400-mb level. In Fig. 2, the profile of L^2 for the combination of the light wind and relatively unstable temperature data exhibits a similar result. However, the L^2 profiles for the light wind condition in combination with the Standard Atmosphere and relatively stable temperature profile do not have as great a slope from 700 mb to the minimum point at 400 mb. Also, the magnitude of L^2 at all levels below 300 mb in these latter cases is definitely larger than the former cases. For all these profiles, the value of L^2 increases steadily above 300 mb. The profiles of L^2 in Figs. 1 and 2 differ somewhat from previously published profiles.[7] The model profiles show that L^2 decreases from 700 to 400 mb, while other published profiles show a decrease to about the 200-mb level. Because of the limited number of levels in the model, important features of the wind and temperature profiles undergo considerable smoothing. The model profiles show the principal features, however, and if more pressure levels had been included the profiles of L^2 would have been more compatible with previous results.

As shown in Fig. 3, the profiles of Ri with strong winds show an increase in Ri from about 850 to above 700 mb, a decrease to about 400 mb, and a gradual increase above this level. The magnitude of Ri for all the strong wind cases is generally less than 10 below the 200-mb level. In Fig. 4, the profiles of Ri for the light wind data in combination with the temperature from the Standard Atmo-

Table 1 Profiles of wind and temperature for model atmospheres

Level, mb	Standard atmosphere height, m	Wind Direction, deg	Wind Speed, m sec^{-1} Strong	Wind Speed, m sec^{-1} Light	Temperature, °C Relatively stable	Temperature, °C Standard atmosphere	Temperature, °C Relatively unstable
850	1457	240	5	4	6.0	5.5	5.0
700	3012	260	12	8	−2.0	−4.7	−6.0
500	5574	265	20	12	−17.0	−21.0	−26.0
400	7185	270	30	18	−27.0	−31.8	−40.0
300	9164	265	42	23	−39.5	−44.7	−57.0
200	11,748	270	25	16	−51.0	−56.6	−60.0
150	13,608	275	15	11	−50.5	−56.6	−64.0
100	16,180	280	7	7	−50.0	−56.6	−67.0

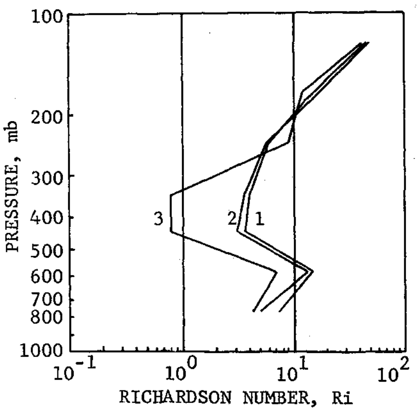


Fig. 3 The profiles of *Ri* for the strong wind profile in combination with selected temperature profiles. (1-Strong wind and relatively stable temperature; 2-Strong wind and standard atmosphere; 3-Strong wind and relatively unstable temperature.)

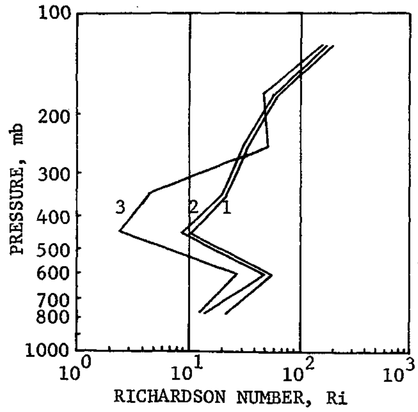


Fig. 4 The profiles of *Ri* for the light wind profile in combination with selected temperature profiles. (1-Light wind and relatively stable temperature; 2-Light wind and standard atmosphere; 3-Light wind and relatively unstable temperature.)

sphere and the relatively stable temperature profile exhibit the same shape as the strong wind cases, but the magnitude of *Ri* is much larger and generally greater than 10. For the case of the light wind and relatively unstable temperature data, an intermediate result occurs for the *Ri* profile. While the general shape of the *Ri* profile is the same as in the strong wind cases, the magnitude of *Ri* is greater than 10 below 500 and above 300 mb, and less than 10 between these levels.

All the strong wind profiles plus the combination of the light wind and relatively unstable temperature data lead to a large decrease of L^2 with height from the top of the mountains to the middle troposphere, and to low values of *Ri*. These conditions are favorable for the development of mountain waves and the occurrence of turbulence. The remaining light wind and temperature combinations show a small decrease of L^2 with height and large values of *Ri*, but these conditions are unfavorable for mountain waves and turbulence. Therefore, the profiles of L^2 and *Ri* for the model profiles are classified as characteristic and noncharacteristic of the development of mountain waves and the occurrence of turbulence.

Data

Aircraft

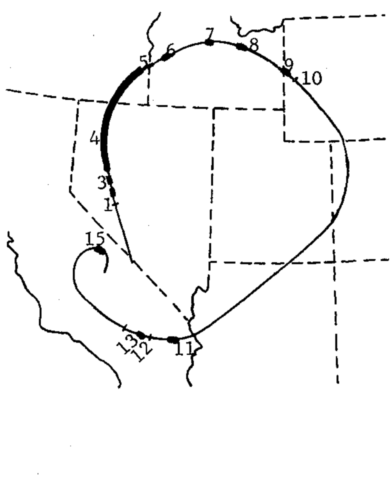
The aircraft data include the flight track, time (all flights were made between 14 and 21GMT, altitude, location, and extent of the turbulence encounters.[15] The encounters were determined from changes in the normal acceleration at the center of gravity of the aircraft. The flight levels for these encounters ranged from 12 to 22 km, and all flights were made over the mountainous regions of the western United States. An example of these data is shown in Fig. 5.

In the 36 cases presented by Ehernberger, each flight had at least one encounter with turbulent conditions. In this paper, a flight with five or less instances of reported turbulence along the flight track is considered as a no-turbulence case, while a flight with six or more turbulence encounters is considered as a turbulence case. These criteria represent a division within the data since all flights classified as turbulence cases had at least eight turbulence encounters along the flight path, vs the five or less encounters for the no-turbulence cases. The results of this classification showed 17 cases of turbulence and 19 cases of no turbulence. Future references to turbulence and no-turbulence cases will be based on this classification.

Meteorological

Rawinsonde data west of 105°W long were used in this study. These data were available at 14 stations twice daily, 00 and 12GMT. Data for both times were used in the analysis; however, the 12GMT data were used more than the 00GMT data.

19 March 1965



Encounter Number	Greenwich Time	Maximum Acceleration g units	Distance in Turbulence n. mi. (km)	Height ft (m)
1	1900Z	0.10	3.4 (6.3)	51.0×10^3 (15.5×10^3)
2	1906Z	.70	20.4 (37.8)	52.0 (15.8)
3	1908Z	.45	13.9 (25.7)	53.6 (16.3)
4	1913Z	1.20	242.9 (449.9)	57.0 (17.4)
5	1919Z	.25	12.1 (22.4)	62.2 (19.0)
6	1921Z	.30	17.2 (31.9)	62.8 (19.1)
7	1923Z	.20	10.2 (18.9)	64.3 (19.0)
8	1925Z	.15	10.7 (19.8)	65.0 (19.8)
9	1930Z	.15	10.0 (18.5)	65.6 (20.0)
10	1931Z	.10	3.1 (5.7)	65.8 (20.1)
11	1958Z	.20	19.7 (36.5)	69.4 (21.2)
12	2000Z	.10	4.2 (7.8)	70.4 (21.5)
13	2001Z	.20	0.9 (12.8)	70.0 (21.3)
14	2003Z	.20	4.5 (8.3)	69.4 (21.2)
15	2015Z	.10	5.6 (10.4)	53.2 (16.2)

Fig. 5 An example of the turbulence data reported by the XB-70 aircraft in the stratosphere (after Ehernberger, 1968).

Analyses of Data

The Normal Wind Component at 700 mb

Numerous mountain ranges in the western United States have crests near the 3-km level. Thus, the 700-mb surface is generally representative of meteorological conditions near mountain-top level. A 700-mb chart was plotted for each XB-70 flight. In addition, wind, temperature, and height data for 500 mb were plotted on each 700-mb chart. The wind component perpendicular to the mountains at mountain-top level, pertinent features of the wind profile in the low and middle troposphere, and the 700- to 500-mb lapse rate of temperature were readily available from these charts.

A normal wind component of 10 m sec^{-1} at mountain-top level was used in this study as the minimum speed required to initiate mountain waves leading to stratospheric turbulence. This speed was chosen on the basis of results presented by Harrison[16,17] and Gazzola.[12]

The wind component normal to the mountain crests was determined from the 700-mb data over the western United States. This wind component and the basic synoptic features of the 700-mb level exhibited an excellent relationship with the occurrence of turbulence in the stratosphere. In eight of nine cases in which the wind component normal to the mountains was greater than 10 m sec^{-1}, the XB-70 encountered turbulence in the stratosphere. In 18 of 22 cases, the normal wind component at mountain-top level was less than 10 m sec^{-1} and no stratospheric turbulence was encountered. In the five remaining cases, a closed low at 700 mb was observed within 5° lat of the XB-70 flight path, and stratospheric turbulence was reported in all five cases. These results are summarized in Table 2. Other data in the table will be discussed later.

The 700- to 500-mb Lapse Rate of Temperature

For each rawinsonde station, the 700- to 500-mb lapse rate of temperature was determined, and these data were analyzed in 2°C intervals. An average value of this lapse rate was determined for each flight track of the XB-70. The average values for this lapse rate of temperature in °C are shown in Table 2 for the turbulence and no-turbulence cases and separated according to the 700-mb features.

Table 2 Synoptic features at 700 mb and the average values along the flight track for the 700- to 500-mb lapse rate of temperature related to the conditions in the stratosphere reported by the XB-70

	Turbulence		No turbulence	
Synoptic features at 700 mb	Total cases	Individual lapse rate values 700–500 mb, °C	Total cases	Individual lapse rate values 700–500 mb, °C
Wind component normal to mountain crests ≥10 m sec^{-1}	8	16,16,16,16, 18,18,18,20	1	20
Wind component normal to mountain crests <10 m sec^{-1}	4	12,14,18,20	18	14,14,16,16,16, 16,16,16,18,18, 18,18,20,20,20 20,22
Closed low within 5° lat of aircraft track	5	14,14,16,18, 20	0	

Table 3 The horizontal gradient of the 700 to 500-mb lapse rate of temperature related to conditions at 700 mb and turbulence reported in the stratosphere by the XB-70

			Horizontal gradient of 700- to 500-mb lapse rate of temperature per 300 naut miles		
Synoptic features at 700 mb	Turbulence	Number of cases	≤2°C	3–4°C	≥5°C
Wind component normal to mountain crests ≥10 m sec^{-1}	Yes	8	0	3	55
	No	1	0	1	0
Wind component normal to mountain crests <10 m sec^{-1}	Yes	4	1	2	1
	No	18	9	9	0
Closed low within 5° lat of aircraft track	Yes	5	0	2	3
	No	0	0	0	0

As an individual parameter, the lapse rate of temperature did not differentiate between cases of turbulence and no turbulence. The lapse rate of temperature in combination with the 700-mb wind data did not improve the results obtained by the normal wind component alone. However, the data seem to suggest that the more stable lapse rates of temperature tend to be associated with the occurrence of CAT in the stratosphere. Perhaps if the temperature data were concurrent with the turbulence reports, the results would be improved.

The Gradient and Advection of Stability

Large horizontal gradients of the 700-to 500-mb lapse rate of temperature were observed to be associated with turbulence cases. A gradient of at least 3 to 4°C per 300 naut miles (560 km) along the flight path was observed in these cases, and gradients as high as 5 to 8°C per 300 naut miles (560 km) occurred also. In no-turbulence cases in the stratosphere, the gradients ranged from 1 to 4°C per 300 naut miles (560 km).

In most cases more stable, rather than less stable, air was advected at tropospheric levels into the areas where turbulence occurred. However, several cases of neutral or less stable advection did not allow any firm conclusions to be made.

The results of this stability analysis are shown in Table 3 and indicate that a horizontal gradient of at least 5°C per 300 naut miles (560 km) for the 700- to 500-mb lapse rate of temperature is associated with turbulence in the stratosphere. A gradient of 2°C or less per 300 naut miles (560 km) is related to nonturbulent conditions in the stratosphere. For gradients of 3 to 4°C per 300 naut miles (560 km), the results are inconclusive since these values are associated with cases of turbulence as well as no-turbulence.

Scorer Parameter and Richardson Number Profiles

The profiles of L^2 and Ri for each XB-70 flight were computed from the reported rawinsonde data from the stations near the flight track of the aircraft. For each of the 36 cases, the L^2 and Ri profiles were averaged for two to five stations, the number of stations depending upon the location and length of the flight track.

The average profiles of L^2 and Ri for turbulence cases which occurred with a normal wind component of at least 10 m sec^{-1} at 700 mb showed strong similarities to the model profiles shown in Fig. 1 and considered representa-

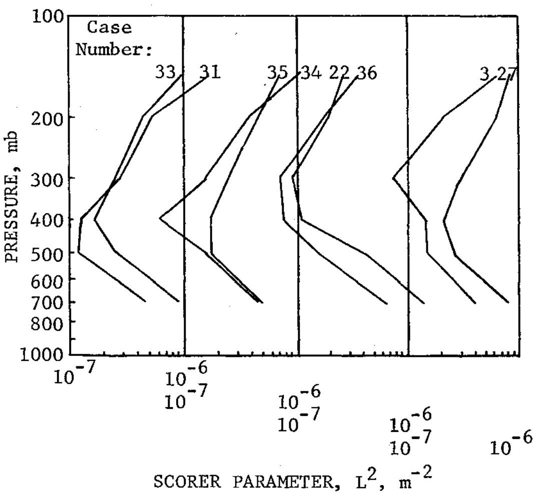


Fig. 6 Eight average profiles of L^2 when the normal wind component at the mountain crests was at least 10 m sec^{-1} and conditions reported in the stratosphere by the XB-70 were classified as turbulence cases.

tive of moutain-wave conditions. Figure 6 shows the average profiles of L^2 for eight cases of this type. In general, these profiles match the model profiles for mountain-wave conditions in shape and magnitude, and they have a minimum value at 400 or 300 mb. Several of these cases show about an order-of-magnitude decrease in the value of L^2 from the mountain-top level to the minimum value in the middle or upper troposphere.

The average profiles of Ri for these eight turbulence cases are shown in Figs. 7 and 8. In general, the profiles display an increase in the values of Ri from the mountain tops to a level in the middle troposphere, a decrease into the upper troposphere, and above this a continuous increase into the stratosphere. These average profiles of Ri show reasonable similarities to the model profiles of Ri for mountain-wave conditions (see Fig. 3).

Examples of average profiles of L^2 are shown in Fig. 9 for cases in which conditions in the troposphere were not favorable for the development of waves. The differences in the shape and magnitude of these profiles in comparison to the mountain-wave cases shown in Fig. 6 are obvious. Since some of these profiles represent turbulence cases, it can be concluded that the turbulence must have been caused by some event other than mountain waves. This conclusion is supported further by the accompanying profiles of Ri shown in Fig. 10. The shape and magnitude of these Ri profiles differ considerably from those of the mountain-wave cases shown in Figs. 7 and 8.

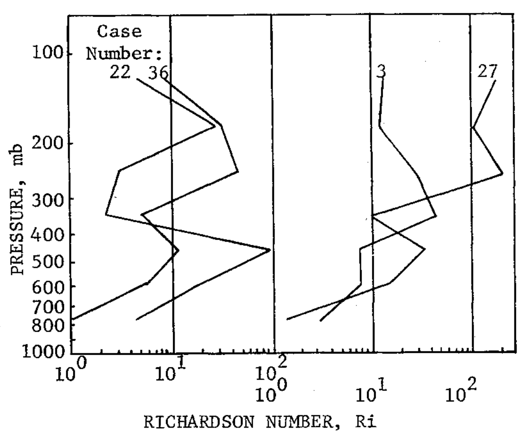


Fig. 7 Average profiles of Ri which correspond to the first four profiles of L^2 in Fig. 6.

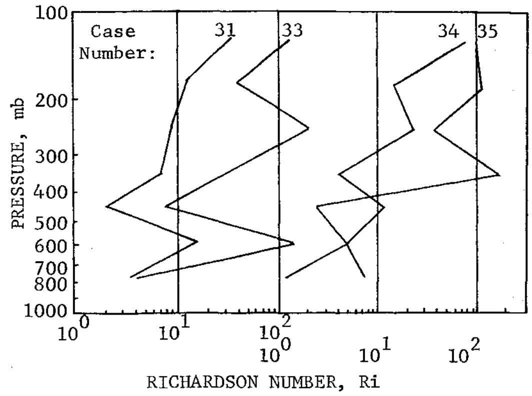


Fig. 8 Average profiles of Ri which correspond to the last four profiles of L^2 in Fig. 6.

Some of the average profiles of L^2 for the no-turbulence cases were similar in shape and magnitude to those associated with mountain waves. Investigation of these cases showed wind profiles with strong north or south wind speeds but without the normal wind component of at least 10 m sec^{-1} at 700 mb. These must be considered as potential mountain-wave cases since only a change in the wind direction would be required to meet the minimum wind criterion of 10 m sec^{-1} normal to the mountain crests.

The most characteristic feature of the model profiles of L^2 and Ri is the minimum point in the middle or upper troposphere. From the model profiles of L^2 and Ri, a minimum value in the middle or upper troposphere was selected that appeared to differentiate between conditions with and without mountain waves. The selected value of L^2 was $3 \times 10^{-7} m^{-2}$, and that for Ri was 10.

For the analysis of the minimum values of the average profiles of L^2 and Ri in the middle or upper troposphere, the 36 cases in this study were grouped according to slightly different criteria than used previously. All the cases with an observed or potential wind component of 10 m sec^{-1} or more and the closed-low cases were grouped together. The remaining cases with observed wind components less than 10 m sec^{-1} normal to the mountain crests comprised the second group of cases. The results of this analysis are shown in Figs. 11 and 12.

For the strong wind and closed-low cases, the minimum profile value of $3 \times 10^{-7} m^{-2}$ for L^2 was exceeded only

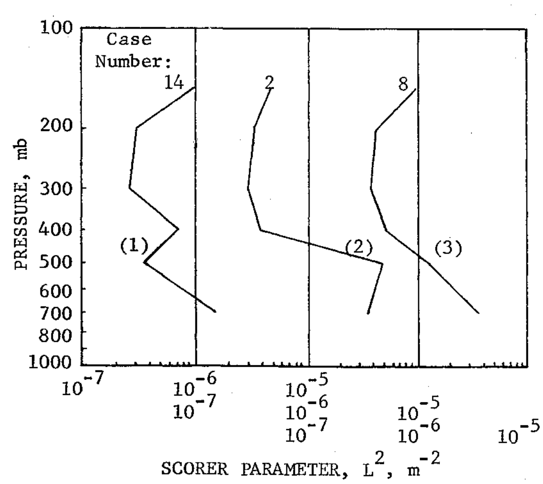


Fig. 9 Average profiles of L^2 for specific conditions at 700 mb and in the stratosphere: 1) a closed low at 700 mb within 5° lat of the flight track of the XB-70 and classified as a turbulence case; 2) the wind component normal to the mountain tops less than 10 m sec^{-1} and classified as a turbulence case; 3) the wind component normal to the mountain tops less than 10 m sec^{-1} and classified as a no-turbulence case.

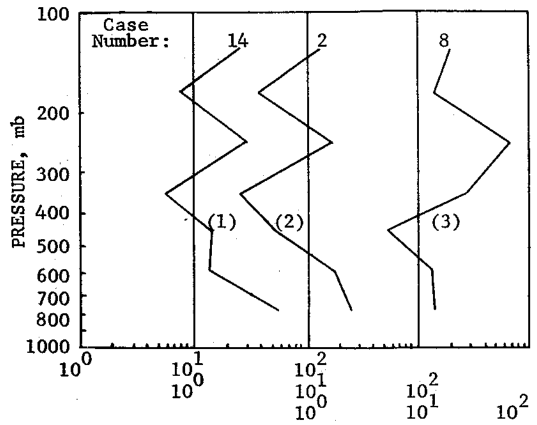


Fig. 10 The average profiles of Ri for the cases and conditions specified for the corresponding profiles of L^2 in Fig. 9.

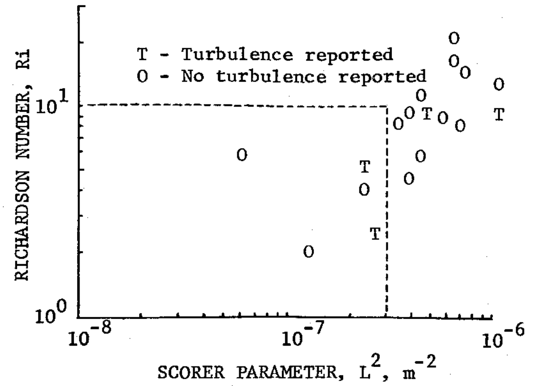


Fig. 12 The minimum value in the middle or upper troposphere of the average profiles of L^2 and Ri for those cases with wind components less than 10 m sec^{-1} normal to the mountain tops. (NOTE: Closed-low cases are not considered in this figure.)

twice, and both of these cases has a minimum value of Ri less than 10. For the light-wind cases, the minimum profile value of L^2 was greater than 3×10^{-7}m^{-2} in 13 of 18 cases. For the five other cases, the minimum value of L^2 was less than 3×10^{-7}m^{-2}, but turbulence was reported in only two of these cases. The minimum value of Ri was greater than 10 in six cases in which no turbulence occurred and the minimum value of L^2 exceeded 3×10^{-7}m^{-2}. For the 12 remaining cases, the minimum profile value of Ri was less than 10.

The magnitude of the minimum profile value for L^2 differentiates between the actual or potential turbulence and no-turbulence cases extremely well, whereas Ri does not. The results are summarized in Table 4. For those cases with an actual or potential wind component normal to the mountain tops and greater than or equal to 10 m sec^{-1}, but excluding closed-low cases, the minimum magnitude of L^2 was less than 3×10^{-7}m^{-2} in 13 out of 13 cases. Turbulence was reported in the stratosphere in 12 of these 13 cases. For the four cases of light winds and reported stratospheric turbulence, 3×10^{-7}m^{-2} was exceeded twice. In the light wind cases with no turbulence, 3×10^{-7}m^{-2} was exceeded in 11 of 14 cases. The closed-low cases were added to the aforementioned results according to wind velocities only. The results are indicated by the figures in parentheses in Table 4.

The minimum profile value of Ri did not produce any conclusive results as far as separating cases of turbulence and no turbulence. Under light wind conditions, no turbulence occurred when the minimum profile value of Ri exceeded 10. In virtually all of the turbulence cases, the minimum profile value of Ri was less than 10; however, a similar result occurred for more than half the cases in which there was no turbulence in the stratosphere. Thus, the Ri value, by itself, does not adequately differentiate between cases of turbulence and no turbulence. Finally, Ri in combination with L^2 does not improve the results obtained by using L^2 alone.

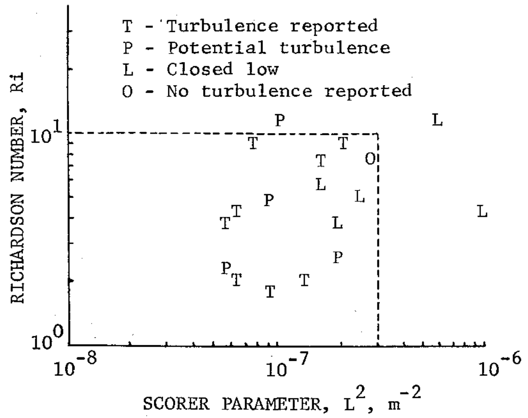


Fig. 11 The minimum value in the middle or upper troposphere of the average profiles of L^2 and Ri for those cases with actual and potential wind components of at least 10 m sec^{-1} normal to the mountain tops and for all closed-low areas within 5° lat of the flight track of the XB-70. (NOTE: The minimum wind criterion was not considered in the closed-low cases.)

Vertical Wind Shear

Vertical wind shear plays an important role in the vertical propagation of wave motions and energy[6,9] as well as in L^2. The terms in Eq. (1) were computed separately, and the ratio of the first to the second term was calculated in order to determine the importance of the shear term to the value of L^2. The results of this analysis are shown in Table 5. For the eight turbulence cases with the minimum wind component of 10 m sec^{-1} normal to the mountain tops, four of 27 stations had no levels with a ratio of the first to the second term of Eq. (1) which was 1:1 or less, eight had one, seven had two, six had three, and two had four levels with this or a smaller ratio.

In general, the magnitude of the wind shear term is important to the value of L^2 for mountain wave and potential mountain wave cases as well as for the closed-low cases. With the minimum wind component of 10 m sec^{-1} normal to the mountain tops, 85% of the stations had at least one level where the ratio of the first to the second term of Eq. (1) was 1:1 or less. However, for the light-wind cases, the shear term displayed a marked decrease in

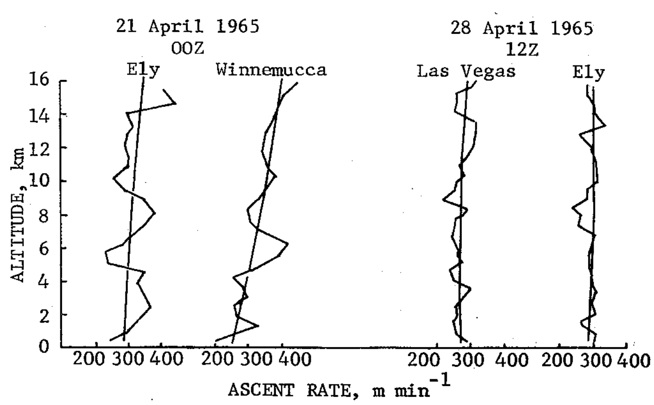


Fig. 13 Examples of the ascent rate of rawinsonde balloons indicative of the presence of mountain waves (21 April 1965) and the absence of such waves (28 April 1965). The smooth lines represent average ascent rate profiles drawn by eye.

Table 4 Turbulence or potential turbulence conditions in the stratosphere related to the minimum value of the average profile of L^2

Actual or potential wind component normal to the mountain crests	Turbulence or potential turbulence L^2 minimum profile value, m^{-2} $<3 \times 10^{-7}$	$>3 \times 10^{-7}$	No turbulence L^2 minimum profile value, m^{-2} $<3 \times 10^{-7}$	$>3 \times 10^{-7}$
≥ 10 m sec^{-1}	12 (2)[a]	0 (1)	1	0
<10 m sec^{-1}	2 (1)	2 (1)	3	11

[a] Closed-low cases in parentheses.

its importance to the value of L^2. The 1:1 ratio or less of the terms in Eq. (1) occurred most frequently near the tropopause. However, the results in Table 5 suggest that the vertical wind shear plays an important role in layers other than just the surface boundary layer and near the jet stream as stated by Foltz.[4]

The magnitude of the vertical wind shear between individual levels compared favorably with the shears in the model profiles for strong winds. Vertical shears of about 8 m sec^{-1} per 2000 m from the mountain tops to the tropopause represent a reasonable minimum value of these shears. The fact that one no-turbulence case occurred with the necessary normal wind component at 700 mb can be attributed to very small vertical wind shears.

Evidence of the Presence of Mountain Waves from the Variation in Ascent Rate of Rawinsonde Balloons

Corby[18] has shown that rawinsonde balloons experience variations in ascent rate when they ascend through wave motions in the atmosphere. Booker[19] found also that a floating balloon experienced sizable up and down motions in the presence of lee waves over central Pennsylvania. Lee waves have been observed on many occasions in the mountainous regions of the western United States, and hence it is logical to expect that these waves would be manifested as changes in the ascent rate of rawinsonde balloons, particularly when the waves are well developed.

Table 5 The ratio of the first to the second term of the Scorer parameter vs the reported conditions in the stratosphere and troposphere

Synoptic features at 700 mb	Conditions in stratosphere	Total cases	Levels with ratio[a] ≤1:1 0	1	2	3	4
		Number of stations with ratio of terms 1:1 or less					
Wind component normal to mountain crests ≥ 10 m sec^{-1}	Turbulence No turbulence	8 1	4 1	8 2	7 0	6 0	2 0
Potential wind component normal to mountain crests ≥ 10 m sec^{-1}	Potential turbulence	4	3	2	3	2	1
Wind component normal to mountain crests <10 m sec^{-1}	Turbulence No turbulence	4 14	9 31	6 12	1 4	0 2	0 0
Closed low within 5° lat of aircraft track	Turbulence No turbulence	5 0	5 0	5 0	7 0	4 0	1 0

[a] Ratio = $\frac{g\beta}{U^2} \Big/ \frac{U''}{U}$.

Vertical profiles of the ascent rate of rawinsonde balloons at a number of locations were computed on days corresponding to the XB-70 flights. The ascent rates were computed from position data at 2-min intervals. Examples of ascent rate profiles are shown in Fig. 13. The first two profiles, computed from data at 00GMT, 21 April 1965, represent conditions favorable for mountain waves, while the two profiles computed from data at 12GMT, 28 April 1965, represent conditions unfavorable for mountain waves. On 21 April 1965, large and cyclic variations in the ascent rates of the balloons were observed, while on 28 April 1965 variations in the ascent rates were much smaller and noncyclic. Profiles displaying these characteristics were found to be typical for each case.

Ascent rate profiles were computed on each date corresponding to the flight of the XB-70, and the data were grouped as discussed previously, i.e., based on the magnitude of the wind component normal to the mountains and the presence of a closed low at mountain-top level. Each profile was analyzed to determine if large amplitude and cyclic variations were present and, if so, this was considered as indicative of the presence of mountain waves. The results are shown in Table 6. The column headed "Case No." in Table 6 has no meaning other than for identification of the flights. A "Yes" is indicated when the ascent rate curves contained large amplitude and cyclic variations similar to those shown in the first two profiles in Fig. 13, and a "No" indicates the absence of these characteristics with the profiles being similar to the last two in Fig. 13. It is clear from Table 6, part a, that when the wind component normal to the mountain ranges is ≥ 10 m sec^{-1}, mountain waves are indicated in most of the ascent rate profiles. As pointed out earlier, turbulence was generally associated with these conditions. Of the profiles examined in this category, only 18% did not show cyclic variations with large amplitude. For the group of data in Table 6, part d, when the wind component normal to the mountains was <10 m sec^{-1}, only 6% of the profiles examined had cyclic variations with large amplitudes. These two categories differentiate quite well between turbulence and no-turbulence conditions. In the case when the wind component normal to the mountains was <10 m sec^{-1} and the aircraft encountered turbulence, only 28% of the profiles indicated the presence of mountain waves. In the closed-low cases shown in Table 6, part e, only 19% of the profiles examined indicated the presence of mountain waves.

Smooth ascent rate profiles were drawn by eye for each rawinsonde sounding considered in Table 6. Examples are shown in Fig. 13. The magnitude of the differences between the average and measured profiles at the maximum and minimum points on the measured profiles were computed. The maximum difference usually was considerably larger than all other values, and was eliminated from further consideration on the basis that it probably resulted from error in the measurements. The average magnitude and range of the near-maximum variations between the measured and average ascent rate profiles are given in

Table 6 Indication of the presence or absence of mountain waves from the variation in ascent rate of the rawinsonde balloon

Case No.	Date	Time (GMT)	Location[a] 1	2	3	4	5	6	7
	a) Turbulence—wind component normal to mountains ≥ 10 m sec^{-1}								
3	20 Apr 65	12	Yes	Yes	Yes				
	21 Apr 65	00	Yes	Yes	Yes				
22	2 Dec 65	00		No		Yes			
	2 Dec 65	12	Yes	Yes		No	Yes		
	3 Dec 65	00	Yes	Yes		No	Yes		
27	3 Jan 66	12		Yes		Yes	Yes		
	4 Jan 66	00				Yes	Yes		
31	9 Feb 66	12			Yes				
	10 Feb 66	00	No		Yes				
33	10 Mar 66	12		Yes		Yes	No		
	10 Mar 66	18					Yes		
	11 Mar 66	00					Yes		
34	15 Mar 66	12		No		Yes			
	16 Mar 66	00		Yes					
35	17 Mar 66	12	Yes	Yes		Yes			
	18 Mar 66	00	No						
36	19 Mar 66	12		Yes					
	20 Mar 66	00		Yes		Yes			
	b) No turbulence—wind component normal to mountains ≥ 10 m sec^{-1}								
7	1 July 65	12	Yes	No			Yes		
	c) Turbulence—wind component normal to mountains <10 m sec^{-1}								
2	4 Mar 65	12	No		No			No	
	5 Mar 65	00	No		No			No	
11	22 Sept 65	12	Yes	Yes	Yes				
	23 Sept 65	00	No	No	Yes		No		
16	2 Nov 65								
	3 Nov 65	00		No			No		
29	12 Jan 66	12	Yes	No					No[b]
	d) No turbulence—wind component normal to mountains <10 m sec^{-1}								
1	16 Feb 65								
4	28 Apr 65	12	No	No	No				
	29 Apr 65	00	No	Yes	No				
8	27 July 65	12	No						
12	30 Sept 65	00	No	No		Yes	No		
13	14 Oct 65	12		No					
15	26 Oct 65								
17	4 Nov 65								
18	12 Nov 65	12	No						
19	18 Nov 65								
20	30 Nov 65	12	No	No	No		No		
	30 Nov 65	18					No		
21	1 Dec 65	00	No	No	No		No		
	1 Dec 65	12		No		No			
23	7 Dec 65	12		No					
	8 Dec 65	00		No					
24	10 Dec 65								
25	11 Dec 65	12	No						
	11 Dec 65	18	No						
	12 Dec 65	00	No						
26	21 Dec 65								
	22 Dec 65	00	No	No					
28	11 Jan 66								
30	15 Jan 66								
32	17 Feb 66	12	No						
	18 Feb 66	00	No	No		No			
	e) Closed low at mountain-top level								
5	7 May 65	12	Yes	No	No				
	8 May 65	00	Yes	No	No				
6	16 June 65	12	No		No		No		
	17 June 65	00			No				
9	18 Aug 65	12							No
	18 Aug 65	18							No
	19 Aug 65	00							No
10	17 Sept 65	12	No		No				
	18 Sept 65	00	No	No	Yes				
14	16 Oct 65	12		No					

[a]Location code:
1 Las Vegas; 2 Winnemucca; 3 Ely; 4 Boise; 5 Salt Lake City; 6 Vandenberg; 7 Oakland
[b]Yucca

Table 7 for each category in Table 6. The average magnitude of the near-maximum variations in ascent rate for the "Yes" cases is about twice as large as that for the "No" cases. As shown in Table 7, the range of these variations is larger for the "Yes" than for the "No" cases.

Turbulence is associated with variations in ascent rate of the rawinsonde balloon (difference between average and measured ascent rate profiles) with near-maximum magnitudes between approximately 50–150 m min^{-1}. This is

Table 7 The average magnitude and range of near-maximum variations in ascent rate of rawinsonde balloons for each category given in Table 6

Category from Table 6	Average magnitude of near-maximum variations in ascent rate, m min^{-1} Yes	No	Range of magnitude of near-maximum variations in ascent rate, m min^{-1} Yes	No
a	74	34	40–180	30–60
b	80	40	[a]	[a]
c	72	43	50–120	30–70
d	85	40	70–100	20–80
e	97	47	80–110	20–70

[a] Insufficient data to establish range.

approximately one-fourth of the average ascent rate which is the value found previously[20] when CAT was observed. The corresponding range for the nonturbulent areas is 30–70 m min^{-1}. In the overlap region between 50–70 m min^{-1}, turbulence cases are characterized by organized and cyclic variations about the average profile, while the nonturbulence cases are characterized by variations which, in most cases, are essentially random. The cyclic variations associated with the "Yes" cases were better organized and observed more often between 3 and 10 km than at higher altitudes, although they were observed on several profiles as high as 25 km.

Forecasting CAT in the Stratosphere from Conditions in the Troposphere

The results of this study have shown positive relationships between conditions in the troposphere favorable for mountain waves and the occurrence of turbulence in the stratosphere. From these results, objective techniques for forecasting CAT in the stratosphere are suggested.

If any of the following conditions exist in the troposphere, turbulence should be forecast in the stratosphere: 1) The normal wind component at the crest of the mountains is greater than or equal to 10 m sec^{-1}, and the wind speed increases with height so as to produce vector wind shears of at least 8 m sec^{-1} per 2000 m; 2) a closed low is present at mountain-top level within 5° of the flight track of the airplane; 3) the horizontal gradient of the 700- to 500-mb lapse rate of temperature is at least 5°C per 300 naut miles (560 km), or; 4) large amplitude, cyclic variations are observed in the ascent of the rawinsonde balloon. From the data analyzed, forecasts based on the first, second, and fourth of the above criteria will verify about 80% of the time. The third criterion verifies 100% when such conditions are observed, but by itself it identifies only 50% of the turbulence cases. For the mountain-wave conditions, turbulence would be expected at distances of several tens of kilometers to the lee of the mountain ranges.

If the normal wind component is at least 10 m sec^{-1} at mountain-top level, and the minimum value of the average profile of L^2 is less than 3×10^{-7}m^{-2}, forecast turbulence in the stratosphere. If the normal wind component is less than 10 m sec^{-1} at mountain-top level, and the minimum value of the average profile of L^2 is greater than 3×10^{-7}m^{-2}, do not forecast turbulence in the stratosphere.

Turbulence in the stratosphere should not be forecast if either of the following conditions occur: 1) The normal wind component at mountain-top level is at least 10 m sec^{-1}, and the minimum value of the average profile of L^2 exceeds 3×10^{-7}m^{-2}, or; 2) the normal wind component at mountain-top level is less than 10 m sec^{-1}, and the minimum value of the average profile of L^2 is less than 3×10^{-7}m^{-2}. In the first case, stability and wind characteristics in the middle and upper troposphere prevent extensive wave development. In the second case, the wind normal to the mountain tops is not strong enough to support development.

Concluding Remarks

From this investigation of turbulence encountered by the XB-70 airplane, there seems to be sufficient evidence to support the hypothesis that there is a direct relationship between conditions in the troposphere favorable for mountain waves and the occurrence of turbulence in the stratosphere. This hypothesis does not explain all the occurrences of stratospheric turbulence; however, mountain waves do appear to be directly related to much of the turbulence in the stratosphere. The profiles of the Scorer parameter (L^2) and Richardson number (Ri) for observed mountain wave conditions agree well with model profiles synthesized to reflect conditions favorable and unfavorable for lee waves. The shapes of the average profiles of these parameters are different for mountain-wave and no-mountain-wave conditions.

The Richardson number computed over layers several thousand feet deep is not a satisfactory parameter to delineate between conditions of turbulence and no turbulence. However, if the minimum value of the average profile of the Richardson number in the middle to upper troposphere exceeds 10, then turbulence in the stratosphere is unlikely. For average values of this parameter less than 10, other criteria are required to reach a conclusion. If the average value of Ri increases from the mountain tops to the middle troposphere, decreases from the middle to upper troposphere, and then increases into the stratosphere, the profile is compatible with the development of mountain waves. The minimum value in the middle or upper troposphere for such a profile should be less than 10.

The normal wind component of at least 10 m sec^{-1} at mountain-top level appears to be the most important of the parameters in the troposphere which were tested as indicators of the probable existence of turbulence in the stratosphere. Also, large horizontal gradients of at least 5°C per 300 naut miles (560 km) for the 700- to 500-mb lapse rate of temperature were observed to be associated with the turbulence cases in the stratosphere. Gradients of 2°C or less per 300 naut miles (560 km) showed a good correlation with the no-turbulence cases. However, the intermediate gradients of 3 and 4°C per 300 naut miles (560 km) were equally representative of turbulence and no-turbulence cases.

The minimum profile value of the Scorer parameter is another important parameter associated with turbulence. If this value is less than $3 \times 10^{-7} m^{-2}$ and the minimum wind component normal to the mountain crests is 10 m sec^{-1} or greater, extensive turbulence can be expected in the stratosphere. In addition, the shape and magnitude of the average profile of the Scorer parameter help to determine the existence of mountain waves.

Vertical wind shear is important in the formation of mountain waves and clear air turbulence. The effects of wind shear, including its derivative as used in the second term of the Scorer parameter, are especially important from the mountain tops to the tropopause, and may be important at other altitudes.

The suggested forecast methods showed about an 80% verification based either upon the normal wind component at mountain-top level and other features of the 700-mb level, the minimum value of the average profile of the Scorer parameter, or the presence of large amplitude, cyclic variations in profiles of the ascent rate of rawinsonde balloons. For such a limited number of parameters, the results are encouraging.

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